Biot-Savart law

(and a few applications)

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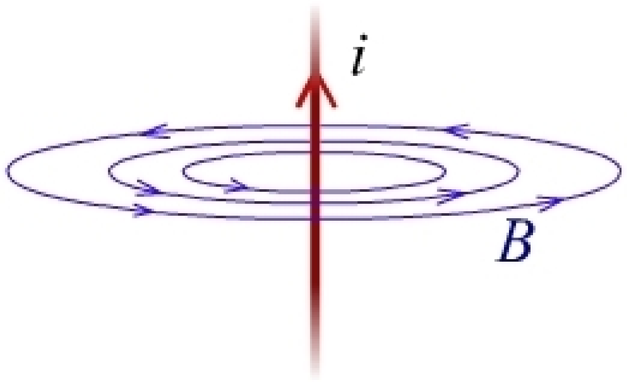
Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Biot-Savart law

(and a few applications)

- Oersted's experiment
- Biot-Savart law
- B due to a circular coil
- B due to a circular arc
- B at an axial point of a coil
- B due to a finite straight wire

Oersted's experiment



Oersted observed the magnetic field due to current carrying conductor. His investigations are significant as they led to the understanding of relation between electric phenomena (current) and magnetic phenomena (magnetic field) through the following observations

- Deflection of the magnetic needle is always tangential to the circle in a plane perpendicular to the wire
- Reversing the direction of current reverses the direction of magnetic field
- Deflection in the magnetic needle increases on increasing the current or bringing the needle closer to the wire

Biot-Savart law

- dl is a small segment of a *real* current carrying wire
- r is distance of the point under consideration from the segment of the wire
- θ is the angle between r and dl (taken in the direction of i)

$$dB \propto i$$

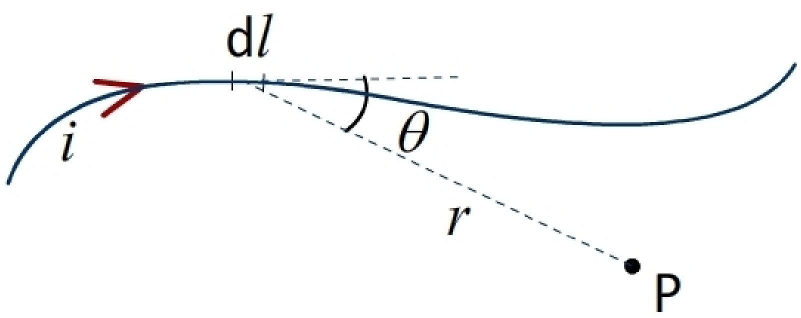
$$dB \propto dl$$

$$dB \propto \frac{1}{r^2}$$

$$dB \propto \sin(\theta)$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dl \sin(\theta)}{r^2}$$

$$\boxed{d\overline{B} = \frac{\mu_o}{4\pi}\frac{i\, d\overline{l} \times \overline{r}}{r^3}}$$



μ_o is called permeability of vacuum

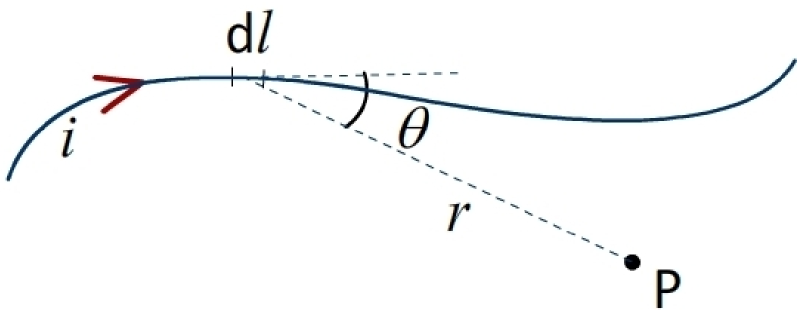
$$\frac{\mu_o}{4\pi} = 10^{-7}$$

Biot-Savart law can be used to calculate the magnetic field induction due to various geometric configurations (straight wire, circular loop, rectangular loop, triangular loop)

Biot-Savart law

When medium between current segment and the point under consideration is not vacuum then the permeability of the medium should be used.

Permeability of a medium (μ) is related to permeability of vacuum or absolute permeability (μ_o) as



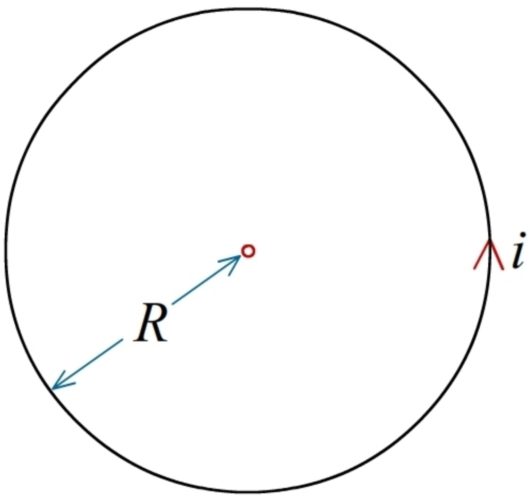
$$\mu = \mu_r \mu_o$$

$$\mu_r = \frac{\mu}{\mu_o}$$

μ_r is called relative permeability

B due to a current carrying circular coil

Consider a circular loop of wire carrying current (i) in counter clockwise direction as shown in the figure. Let R be radius of the circular segment.



Any point on the circular wire is at the same distance from the centre. At any point, angle between dl ,taken in the direction of current, and the radius vector is 90°.

Magnetic field at the centre, due to any segment of the wire, is directed outwards

Using Biot-Savart law

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,\sin(\theta)}{r^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,\sin(90)}{R^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl}{R^2}$$

Integrating over the entire length of the coil

$$B = \int_0^{2\pi R}\frac{\mu_o}{4\pi}\frac{i\,dl}{R^2}$$

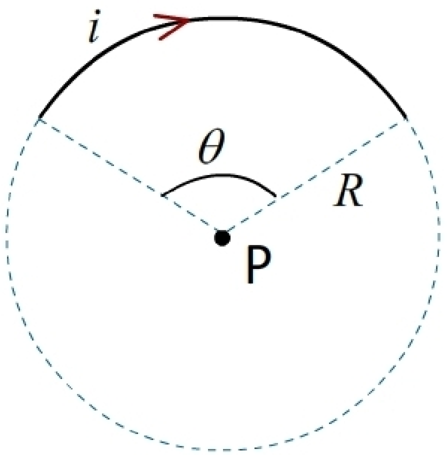
$$B = \frac{\mu_o}{4\pi}\frac{i}{R^2}\int_0^{2\pi R} dl$$

$$B = \frac{\mu_o i}{2R}$$

For n number of turns in the coil we get

$$\boxed{B = \frac{\mu_o n i}{2R}}$$

B due to a current carrying circular arc



Consider a segment of circular loop of wire carrying current i in clockwise direction as shown in the figure. Let R be radius of the circular segment.

Any point on the circular wire is at the same distance from the centre. At any point, angle between dl ,taken in the direction of current, and the radius vector is 90°.

Magnetic field at the centre, due to any segment of the wire, is directed outwards

Using Biot-Savart law

$$\mathrm{d}B = \frac{\mu_o}{4\pi} \frac{i\, dl \sin(\theta)}{r^2}$$

$$\mathrm{d}B = \frac{\mu_o}{4\pi} \frac{i\, dl}{R^2}$$

Integrating over the given arc we get

$$B = \int_0^\theta \frac{\mu_o}{4\pi} \frac{i\, R\mathrm{d}\theta}{R^2}$$

$$B = \frac{\mu_o}{4\pi} \frac{i}{R} \int_0^\theta \mathrm{d}\theta$$

$$B = \frac{\mu_o}{4\pi} \frac{i\theta}{R}$$

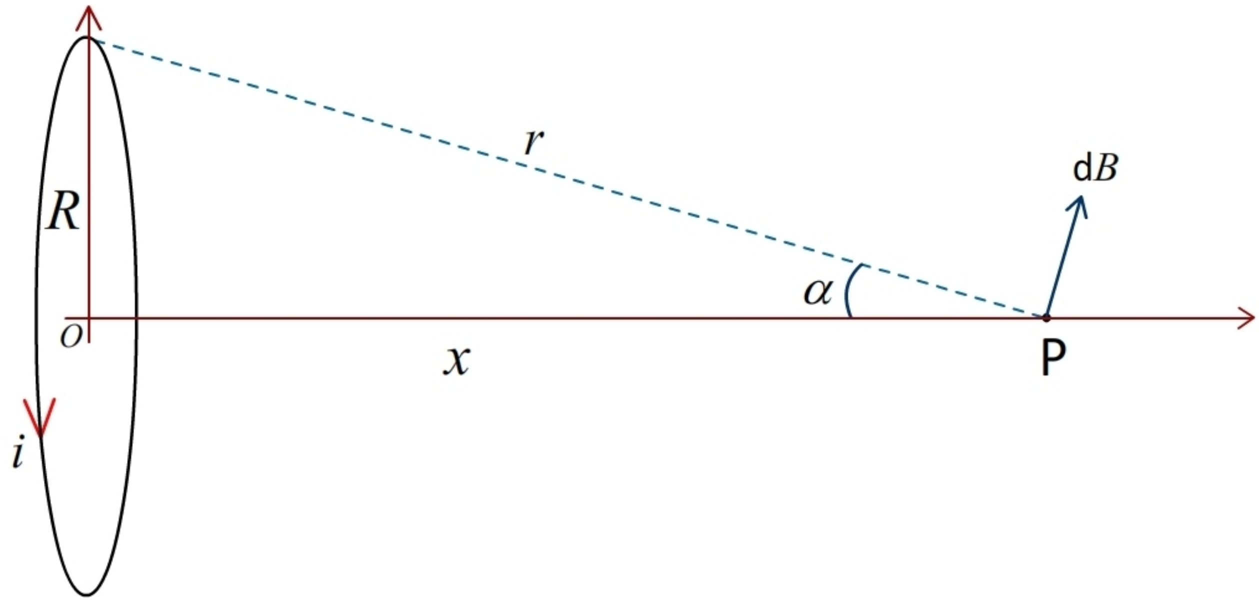
In terms of magnetic field due to a circular loop (B_o)

$$B = \frac{\mu_o i}{2R} \frac{\theta}{2\pi}$$

$$\boxed{B = B_o \left(\frac{\theta}{2\pi} \right)}$$

B due to a current carrying circular coil – at a point on the axial line

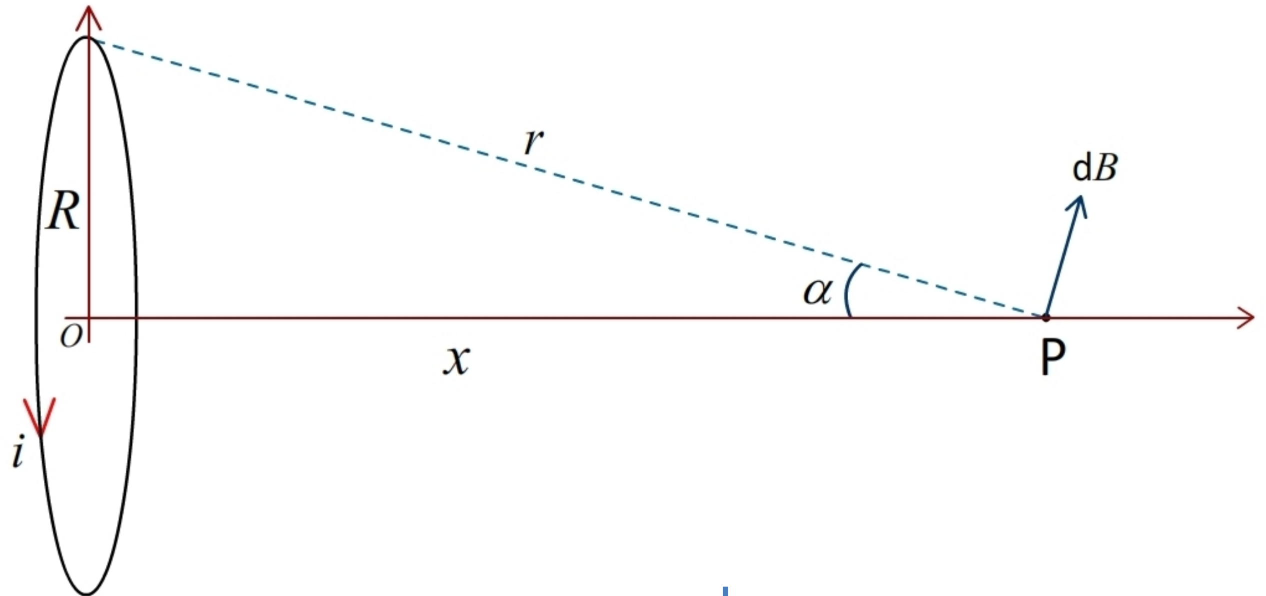
Consider a wire carrying current i. Let the wire be in the form of a circular loop of radius R. Let P be a point at a distance x from the centre of the loop.



Any point on the circular coil is at the same distance (r) from the point P. At any point, angle between the small segment of the wire (dl) taken in the direction of current and the radius vector (r) is 90^o.

Magnetic field at the point may be resolved into two mutually perpendicular components. Components along the axial line get added up and components perpendicular to the axial line get cancelled

B due to a current carrying circular coil – at a point on the axial line



Using Biot-Savart law

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,\sin(\theta)}{r^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl\,\sin(90)}{r^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\,dl}{\left(R^2 + x^2\right)}$$

$$dB_x = \frac{\mu_o}{4\pi}\frac{i\,dl}{\left(R^2 + x^2\right)}\sin(\alpha)$$

$$dB_x = \frac{\mu_o}{4\pi}\frac{i\,dl\;R}{\left(R^2 + x^2\right)^{3/2}}$$

Total magnetic field is obtained by integrating

$$B = \frac{\mu_o}{4\pi}\frac{iR}{\left(R^2 + x^2\right)^{3/2}}\int_0^{2\pi R} dl$$

$$B = \frac{\mu_o}{4\pi}\frac{iR\,2\pi R}{\left(R^2 + x^2\right)^{3/2}}$$

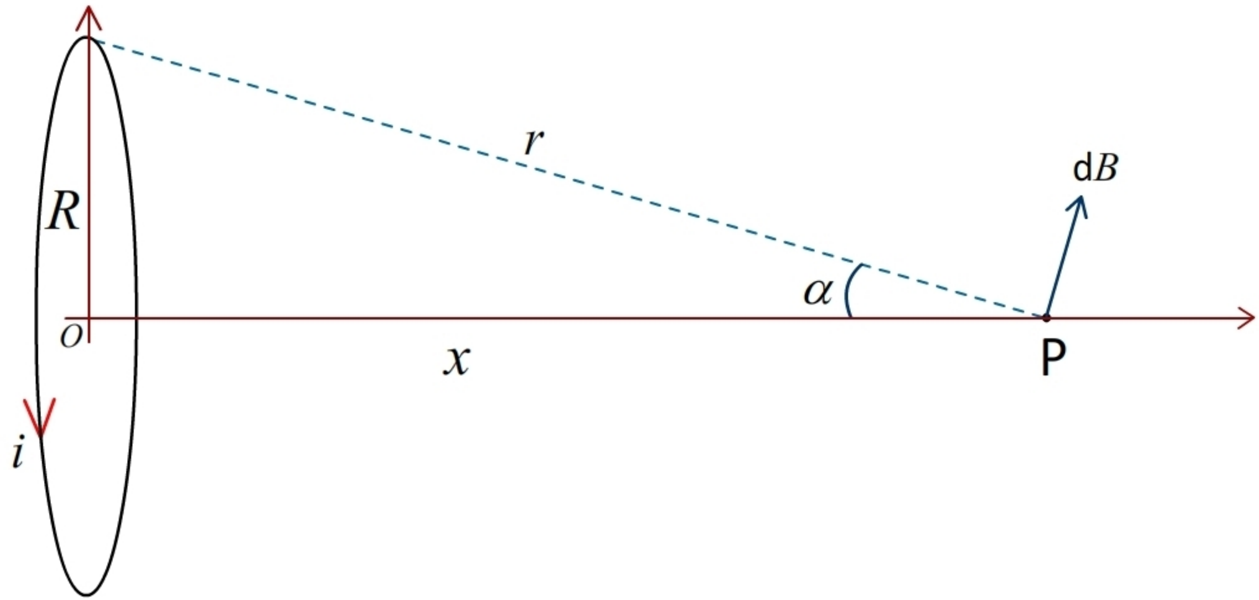
$$B = \frac{\mu_o}{2}\frac{iR^2}{\left(R^2 + x^2\right)^{3/2}}$$

For n turns of the coil

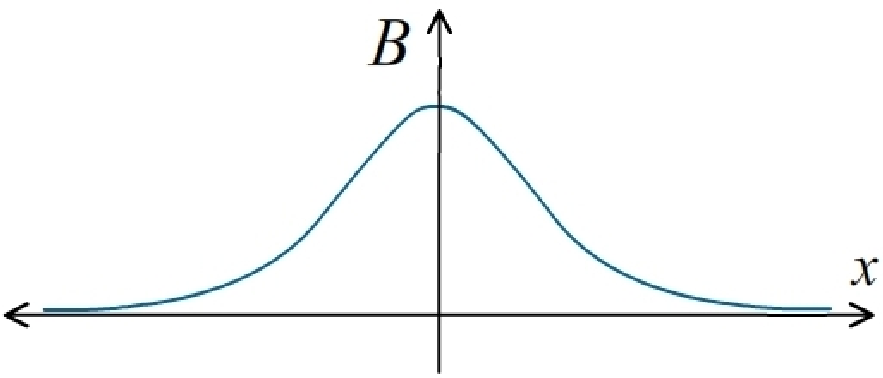
$$\boxed{B = \frac{\mu_o}{2}\frac{niR^2}{\left(R^2 + x^2\right)^{3/2}}}$$

B due to a current carrying circular coil – at a point on the axial line

$$B = \frac{\mu_o n i R^2}{2\left(R^2 + x^2\right)^{3/2}}$$

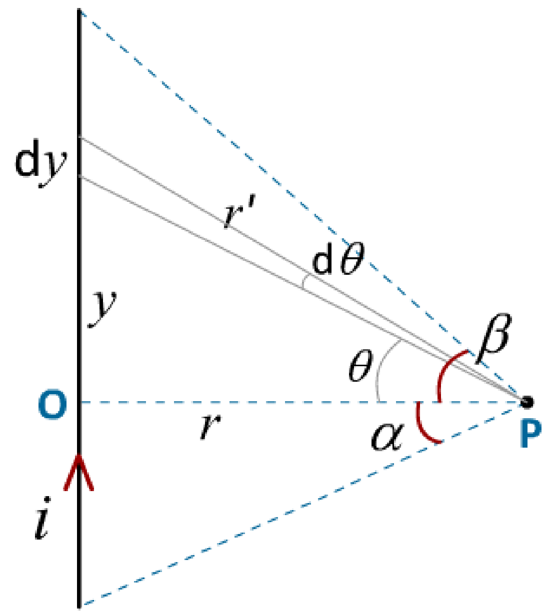


- Magnetic field is the maximum at the centre
- Magnetic field strength tends to zero for large values of x
- Direction of magnetic field remains same on either sides of the coil



B due to a thin straight current carrying wire

- Current in the wire is i
- Perpendicular distance of point P from the wire is r
- dy is a small segment of current carrying wire
- dθ is the angle subtended by segment dy at point P
- y is the height of dy from O
- α and β are the angles subtended by end points of the wire w.r.t. the normal drawn to the wire
- Magnetic field due to any segment of the wire is inwards at point P



- Distance of any segment dy of the wire is NOT same from point P
- Angle between dy and r' is NOT same for any segment considered
- Mathematical technique of integration is required to obtain the resultant B

B due to a thin straight current carrying wire

Using Biot-Savart law

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dl \sin(\theta)}{r^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dy \sin(90+\theta)}{(r')^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dy \cos(\theta)}{(r')^2}$$

From the figure we get

$$\cos(\theta) = \frac{r}{r'}$$

$$r' = \frac{r}{\cos(\theta)}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dy \cos(\theta)}{(r/\cos(\theta))^2}$$

$$dB = \frac{\mu_o}{4\pi}\frac{i\, dy \cos^3(\theta)}{r^2}$$

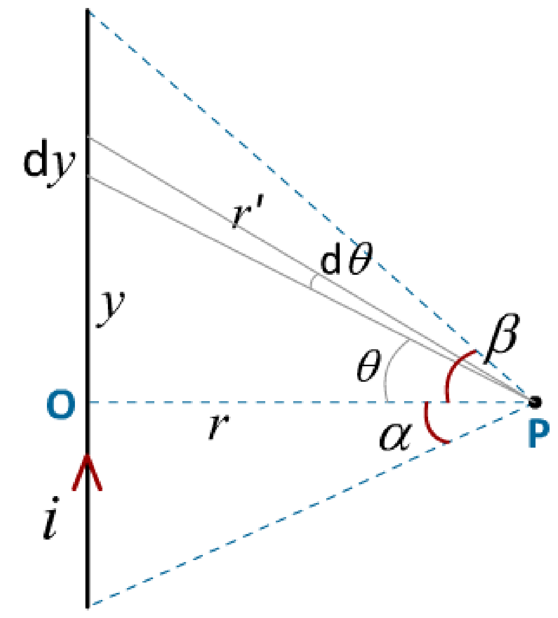
From the figure we get

$$\tan(\theta) = \frac{y}{r}$$

$$dy = r \sec^2(\theta)\, d\theta$$

substituting in above eq we get

$$dB = \frac{\mu_o}{4\pi}\frac{i \cos(\theta) d\theta}{r}$$



Total magnetic field is obtained by integration

$$B = \frac{\mu_o}{4\pi}\frac{i}{r}\int_{-\alpha}^{+\beta}\cos(\theta)\, d\theta$$

$$\boxed{B = \frac{\mu_o}{4\pi}\frac{i}{r}\left[\sin(\alpha)+\sin(\beta)\right]}$$

General approach to determine *B* due to a given geometrical configuration.

- Theoretically break up the given configuration into different segments.
- Calculate the magnetic field at the given point due to each segment
- Use standard relations of the method of integration for the above step
- Magnetic field due to any segment that lies along the line joining it to the given point is zero (because $\sin(0^o) = \sin(180^o) = 0$)
- Use principle of superposition to determine the net magnetic field at the given point as the vector sum of the contributions as mentioned above